3) $n \in \mathbb{Z}$ given 0 is a unit mod n, prove $n = \pm 1$

$\rightarrow$ 0 unit mod n means $\gcd(0,n) = 1$

$0x + ny = 1$ $\quad x, y \in \mathbb{Z}$

$0 \cdot 1 + n \cdot 0 = 0$

$0 \cdot 0 + n \cdot 1 = n$

$0 + yn = 1$

can only be true if

$n = 1$ or $n = -1$

$y = 1$ $\quad y = -1$ so $n = \pm 1$

$\leftarrow$ given $n = \pm 1$, prove 0 is a unit mod n

take mod 1

all the numbers in mod 1 are $\{\bar{0}\}$

and $\bar{0} \cdot \bar{0} = \bar{0} = \bar{1}$ so it is a unit of mod 1

mod -1 = mod 1 so the same can be said of mod -1 as it is equal

definition of a unit: unit times its inverse is $\bar{1}$ mod n

$\bar{0} = \bar{1}$ only in mod ± 1

both directions are proven, the statement holds

Egge Rouwhorst S1737496

1) $7 \bmod 42 \cdot (13 \bmod 42)^{-1}$ $\mathbb{Z}/42$

$\bar{7}\cdot\bar{13}^{-1} = \bar{7}\cdot\bar{13} = 91 \bmod 42$ $\quad 13^{-1}$ is a unit mod 42

$= \bar{7}$ so $13x + 42y = 1$ $x, y \in \mathbb{Z}$

$13\cdot 0 + 42\cdot 1 = 42$

$13\cdot 1 + 42\cdot 0 = 13$

$13\cdot -3 + 42\cdot 1 = 3$

$13\cdot 13 + 42\cdot -4 = 1$

$x = 13 \wedge y = -4$

$13^{-1} \bmod 42 = \bar{13}$

2) $2^1+5=7$ say $9 \mid 2^{6n+2}+5$ for $n \in \mathbb{N}$ (and $n \geq 0$

$9 \mid 2^2+5 = 9 \leftarrow n=0$ □

$9 \mid 2^8+5$ assume $9 \mid 2^{6k+2}+5$ holds for integer k $k \geq 0$

$9 \mid 2^{14}+5$ prove $k+1$ holds too

$2^{6(k+1)+2}+5 \overset{?}{=} 9g$ with $g \in \mathbb{Z}$

$2^{6k+6+2}+5 \overset{?}{=} 9g$

$64\cdot 2^{6k+2}+5 \overset{?}{=} 9g$

$64(2^{6k+2}+5) - 315 \overset{?}{=} 9g$ which is true as $9 \mid 64(2^{6k+2}+5)$ by assumption and $9 \mid -315$

□

so $9 \mid 2^{6n+2}+5$ for $n \in \mathbb{N}$

$24092018 \overset{?}{=} 6n+2$ if this is true then that what has to be proven is also true

$240922016 = 6n$

$n = 401536$ $(6 \mid 240922016)$

~~and as there is an~~

therefore we know that $9 \mid 2^{240092018}+5$